



Improving the robustness of cosmological parameter estimation by modeling emulator residuals with gaussian process regression

Shuto Hoshi, Kazuharu Bamba
Fukushima University

In the current era of precision cosmology, machine-learning-enabled emulators are widely used to achieve fast and accurate estimation of cosmological parameters. CosmoPower is one such emulator that achieves high-precision approximations within a limited parameter space. However, next-generation observations like CMB-S4 reduce observational errors, so the emulator's approximation errors may no longer be neglected as one of the sources of theoretical uncertainty. Therefore, this study focuses on emulation errors, quantitatively evaluates their impact, clarifies their potential effects on inference results, and assesses the reliability of the emulator for next-generation observations.

Introduction

- In the current era of precision cosmology, calculations of cosmological observables based on Einstein–Boltzmann codes are essential, but the computational cost of analyzing large amounts of data poses a serious challenge. In response to this, the machine-learning-enabled emulator CosmoPower was developed. It uses predicted observables from theoretical calculation codes to train a neural network that can then output observables directly from cosmological parameters. CosmoPower is gaining attention as a method capable of rapidly predicting observables with an accuracy comparable to that of theoretical calculation codes. Applications that solve the inverse problem of estimating cosmological parameters from observables are also being explored.

- Meanwhile, next-generation observations such as those from CMB-S4 are expected to significantly reduce observational errors. However, if observational errors become smaller than current theoretical prediction errors, the emulation error arising from predictions of observables by CosmoPower could potentially impact the solution of inverse problems such as cosmological parameter estimation. Therefore, it is crucial to quantitatively evaluate this emulation error.

- This study models the emulation error from CosmoPower using Gaussian process regression and evaluates its impact on cosmological parameter estimation. [A. Spurio Mancini et al., MNRAS **511**, 1771 (2022)]

Methodology

Mock CMB power spectra (TT/TE/EE) computed with CLASS assuming Planck 2018–consistent cosmological parameters are used to isolate emulator errors.[D. Blas et al., JCAP **07**, 034 (2011); N. Aghanim et al. (Planck Collaboration), A&A **641**, A6 (2020)]. Cosmological parameters are inferred using the observational covariance matrix, with priors restricted to the emulator's valid range

Parameter	CMB emulator Range
ω_b	[0.0215, 0.023]
ω_{cdm}	[0.11, 0.13]
h	[0.66, 0.7]
n_s	[0.955, 0.975]
τ	[0.045, 0.065]
$\ln(10^{10} A_s)$	[2.95, 3.15]

The emulation error in CosmoPower predictions is explicitly modeled using Gaussian Process Regression (GPR).

$$\Delta C_\ell^{\text{GPR}} \sim \mathcal{N}(\mu_{\text{GPR}}(\theta), \Sigma_{\text{GPR}}(\theta)).$$

μ_{GPR} : Mean residual estimated by GPR
 Σ_{GPR} : Covariance matrix of the emulation error

The likelihood function $\mathcal{L}(d|\theta)$ assumes a multivariate Gaussian distribution.

$$-2 \ln \mathcal{L}(d|\theta) = (\mathbf{d} - \mathbf{t}(\theta))^T \mathbf{C}^{-1} (\mathbf{d} - \mathbf{t}(\theta))$$

$\mathbf{d}$: $\mathbf{d}_{\text{obs}}$ denotes the power spectrum of the mock data.
 $\mathbf{C}$: Covariance matrix of observational errors
 $\mathbf{t}(\theta)$: Theoretical prediction

The GPR-estimated mean and uncertainty, μ_{GPR} and, Σ_{GPR} are incorporated into the likelihood, and four cases are compared.

Case	$\mathbf{t}(\theta)$	$\mathbf{C}$
CP	C_ℓ^{CP}	C_{obs}
CP+Mean	$C_\ell^{\text{CP}} + \mu_{\text{GPR}}$	C_{obs}
CP+Cov	C_ℓ^{CP}	$C_{\text{obs}} + \Sigma_{\text{GPR}}$
CP+Mean+Cov	$C_\ell^{\text{CP}} + \mu_{\text{GPR}}$	$C_{\text{obs}} + \Sigma_{\text{GPR}}$

C_ℓ^{CP} : Theoretical Power Spectrum by CosmoPower. [A. Spurio Mancini et al., MNRAS **511**, 1771 (2022)]

C_{obs} : the observational covariance matrix errors constructed from the mock data

The posterior distribution is defined as

$$P(\theta|d) \propto \mathcal{L}(d|\theta) \pi(\theta).$$

The impact of theoretical error on the posterior is evaluated by Markov chain Monte Carlo (MCMC) sampling using Cobaya, a widely used Bayesian inference code for cosmological parameter estimation.

[J. Torrado and A. Lewis, JCAP **05**, 057 (2021)].

Results and Discussions

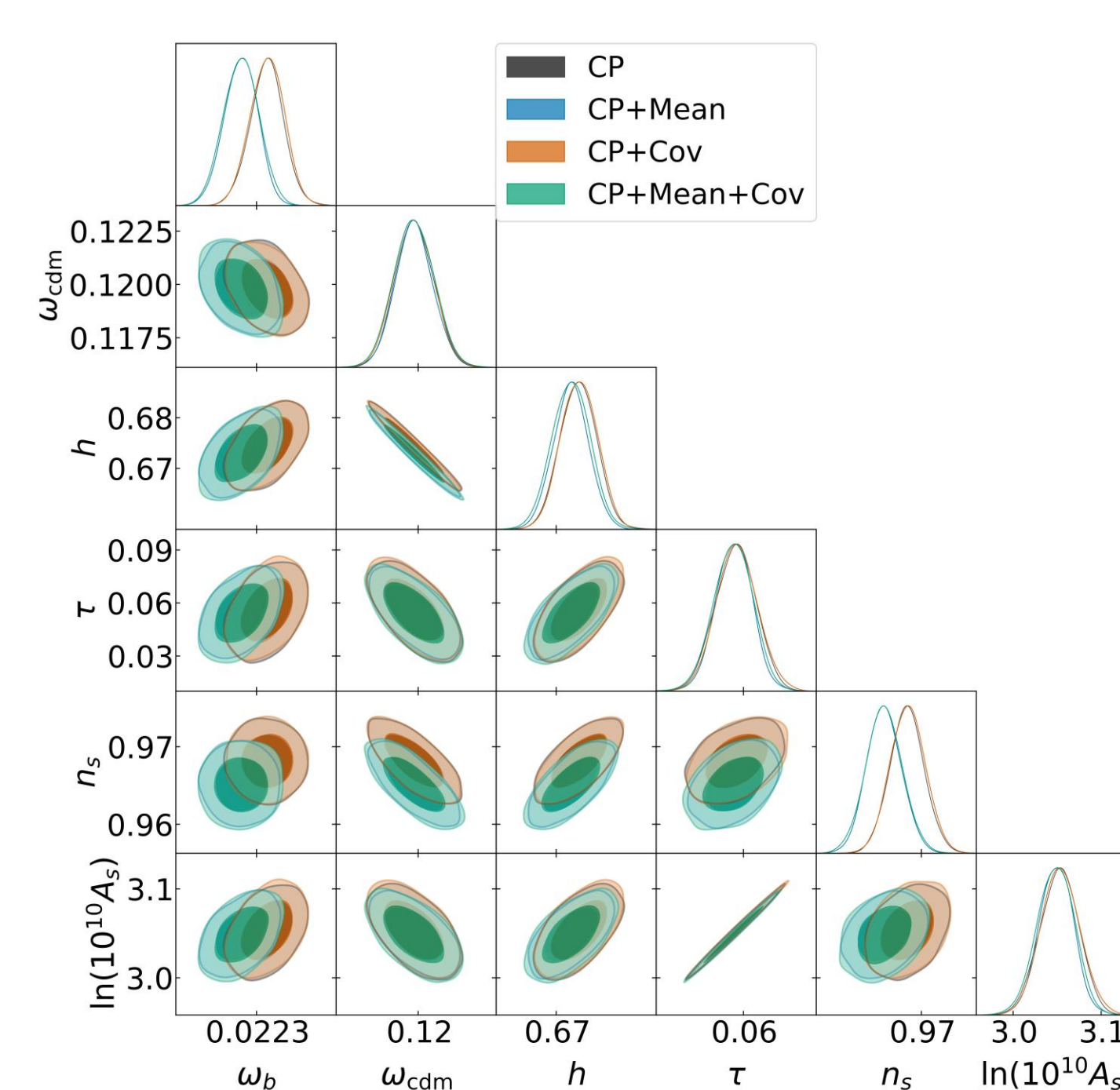


Figure 1: Corner plot showing the posterior distributions and their correlations, obtained from CLASS-based mock CMB data. The contour lines represent the 68% and 95% credible intervals.

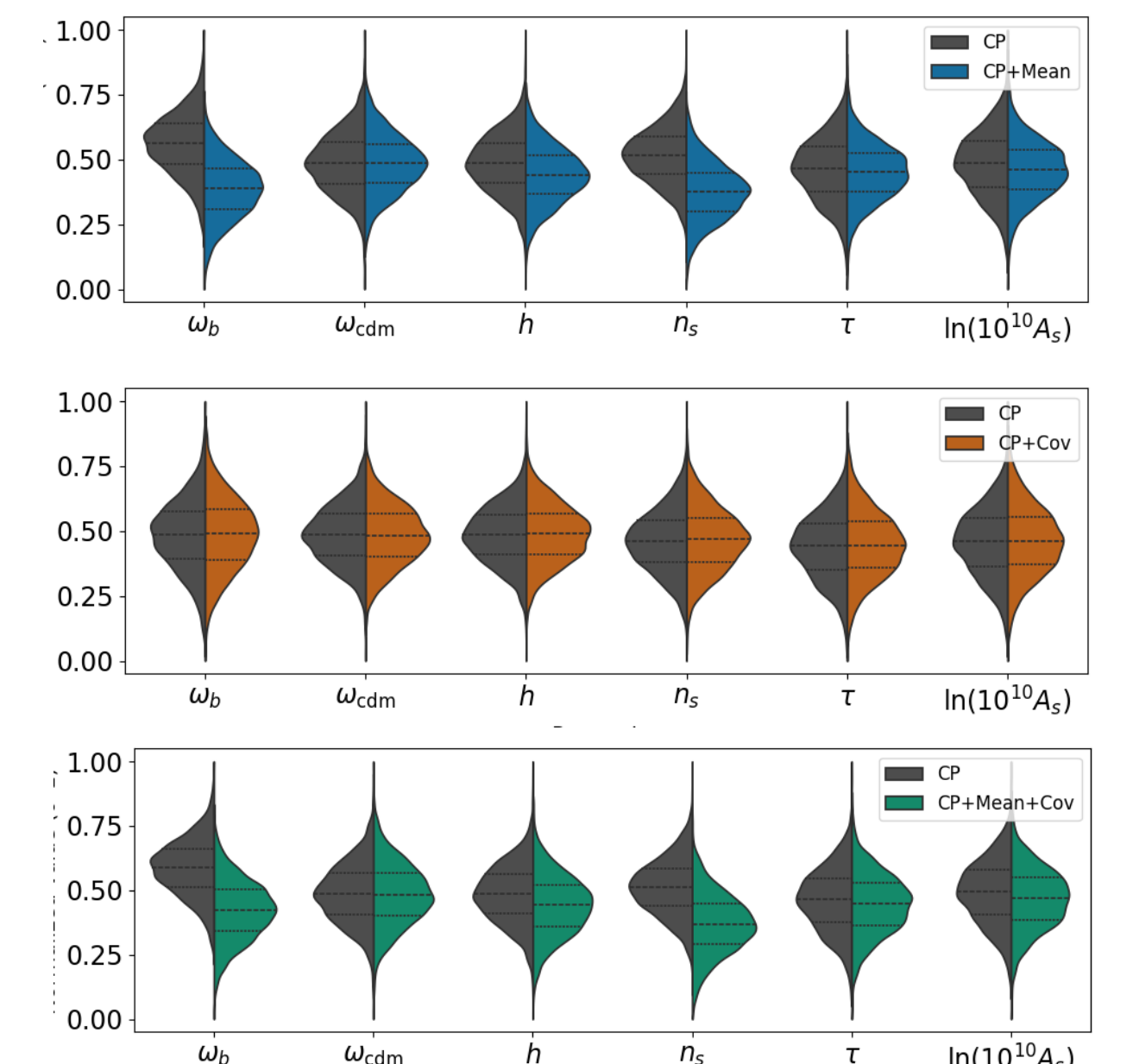


Figure 2: Normalised violin plots comparing the posterior distributions of the baseline case and analyses including emulation errors, illustrating shifts in the central values and changes in the credible intervals.

Adding the GPR-predicted mean error to the theoretical prediction leads to a systematic shift in the posterior central value, whereas treating the error as an additional covariance preserves the central value while broadening the credible intervals. The magnitude of these effects depends on the parameter, with the strongest impact spectral index n_s .

Conclusions

We have evaluated the impact of emulation errors inherent in CosmoPower theoretical predictions on cosmological parameter estimation. The results showed that, although the mean error shifts the central value of the posterior distribution, the covariance matrix of errors only expands the credible intervals while maintaining the central intervals. This study highlights the importance of statistically accounting for theoretical errors in next-generation observations.